Modeling for plug-and-play control in strongly coupled nonlinear networks

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1. Introduction
Many real-world network systems (electric power grids, coupled robotic systems, biological systems) are becoming more strongly coupled than in the past; coupling is both temporal and spatial. This raises a basic question as to whether coupling can be used for cooperative control. This path rules out many existing methods, such as temporal decompositions using standard singularly perturbed forms for near-optimal composite control design; it also rules out the use of non-standard singularly perturbed forms for deriving family of models and corresponding control designs for simpler network models. To explore this question and its implications on complexity and performance of control/communication designs, we use our recently introduced mechanical system representation of interconnected electric power grid; w.l.g we consider potential of controllers in the nodal components of the network, as well as potential of fast switched control of its branch components. We pose the open problem of modeling and controlling such networks. Particular emphasis is on physics-based interpretation of the new models and the resulting controllers. The general underlying question is the best state space choice for multi-layered/plug-and-play control.

2. Description of the Problem
Consider a two-component nonlinear power network shown in Fig. 1. The network comprises two dynamical components, generator and its control (governor and exciter); and a transmission/distribution line and its controller (FACTS). Its mechanical representation was recently proposed [1,2]; it comprises a rotating pendulum, mounted on a moving mass M2, and mass M2 sliding on another mass M3. These represent rotor shaft, rotor winding and stator winding of a typical AC generator, respectively. The transmission line with a controlling FACTS is represented as a spring whose stiffness is controlled by connecting and disconnecting it to the rest of the system; mass Mb is the inerter connected in parallel with the controlled spring. The torque control of the rotor shaft is denoted as force Fp and the translational force pushing M2 represents the DC voltage source in the rotor winding of the generator is Fc. The resulting speed of M3 represents the stator winding voltage. Note that M3 can be neglected in a system with single AC generator; the models presented here are fully generalizable with M3 included [1,2].
The open problem is model of the interconnected mechanical network shown in Fig. 2 and the control design for three controllable inputs [Fp, Fc, and K].

The standard state space model is

\[
\begin{align*}
\dot{X}_A &= f(X_A, X_B, u_A) \\
\dot{X}_B &= f(X_B, X_A, u_B)
\end{align*}
\]

where \( u_A = \begin{bmatrix} F_p & F_c \end{bmatrix} \) and \( u_B = \begin{bmatrix} K \end{bmatrix} \).

Given the strong coupling between components (modules) A and B, it is challenging to design MIMO controller so that: 1) pendulum rotates at a desired frequency, and, 2) modules A and B are synchronized and states are within the given limits.

Acceleration as the coupling (interaction) variable of modules A and B \([2,3]\).

Stored energy and its rate of change as the coupling (interaction variables) between modules A and B \([4]\).

Assuming potential energy zero when pendulum is inverted, the new state space takes on the form

\[
\begin{align*}
\dot{X}_A &= \begin{bmatrix} \dot{\theta} & \omega & \dot{v}_2 \end{bmatrix} \\
\dot{X}_B &= \begin{bmatrix} \dot{\theta} & -\omega & \dot{v}_2 \end{bmatrix} \\
\end{align*}
\]

\[
U = \begin{bmatrix} u_A & u_B \end{bmatrix} \quad u_A = \begin{bmatrix} F_p & F_c \end{bmatrix} \quad u_B = \begin{bmatrix} K \end{bmatrix}
\]

Interaction variables-based multi-layered model of strongly coupled networks shown in Figs. 1 and 2:

\[
\begin{align*}
\dot{X}_A &= f_A(\overline{X}_A, Z_A, P_A, u_A) \\
\dot{Z}_A &= f_{ZA}(\overline{X}_A, Z_A, P_A) \\
\dot{P}_A &= f_{PA}(\overline{X}_A, P_A, \dot{P}_A)
\end{align*}
\]

\[
\begin{align*}
\dot{Z}_B &= f_{ZB}(Z_B, \dot{P}_A) \\
\dot{P}_B &= f_{PB}(P_B, \dot{P}_A)
\end{align*}
\]

Detailed derivations show several striking differences between using state space which stresses the role of acceleration in coupled networks, when compared to the model which uses stored energy and rate of stored energy as interaction variables. Most generally, model above which uses acceleration is only approximate. It requires that the effect of centrifugal force on mass M2 be negligible; this is only true around stable pendulum position. One open question is whether this model can then be used for controlling the coupled network so that the relative position of the pendulum (measured w.r.t. desired synchronous speed) is position of the inverted pendulum. Our conjecture is that this is not possible in this state space. This model also requires explicit measurement or ideal acceleration controller in order to control interactions and internal dynamics as multiple layers. On the other hand, when stored energy and its rate of change (power) are used explicitly as interaction variables, no approximations are required. It becomes possible to stabilize this network at the inverted pendulum position, much the same way as in [5]. In relation to [5], actuator dynamics is not neglected in the model presented here.

3. History and Motivation of the Problem

To the best of our knowledge, the problem formulated here is not explicitly stated in the control nor power systems literature. Much of the work on smart grids starts by using so-called droop
characteristics of the generators (linearized approximate transfer function between frequency set point and power generated; and, between terminal voltage $V$ of wind power plant and the reactive power $Q$). Interconnected power grids are beginning to experience so-called subsynchronous control instabilities (SSCI) which are basically unstable interactions between fast FACTS controllers on power plants (such as DFIG on wind power plants and the FACTS control of weak transmission lines [6]). Open questions are numerous regarding control of such fast interactions. There have been only very few recent papers recognizing the problem of SSCI; the problem is very real and it requires major attention. In reference to Figs. 1 and 2, how does one decide relative relevance of mechanical force, electrical energy applied to the rotor winding and very fast power electronically shaped stiffness of the inerter?

Finally, we believe that the problem of extending rich literature on nonlinear control of stand-alone components to the strongly connected nonlinear components of dynamic networks (natural dynamics and embedded control in both nodes and branches) is the broad open problem. Most of the existing literature supports modular approach to competitive control. However, in real world networks it remains critical to meet objectives without assuming that each dynamic component has a controller. The role of cooperative control is increasingly important for these reasons [6].

Plug-and-play specifications could be related to specifications for connecting new (group of) modules so that the augmented system meets the performance objectives. Fundamentally, this problem become the one of information exchange for performance specifications at various layers within a strongly coupled network. This is a much harder problem than the problem of decentralized control in weakly connected networks.

**Bibliography**
